Unruh Effect with Back-Reaction - A First Quantized Treatment

B. Reznik*

T-6 MS B288, Los Alamos National Lab., Los Alamos, NM 87545

Abstract

We present a first quantized treatment of the back-reaction on an accelerated particle detector. The evaluated transition amplitude for detection agrees with previously obtained results.

*e-mail: reznik@t6-serv.lanl.gov

I. INTRODUCTION

In view of the close connection of black hole radiation and acceleration radiation [1-3] it is reasonable to expect that some of the difficulties regarding the former case, could be mirrored and examined on the latter simpler problem. In particular, it can be anticipated that the modifications of the Unruh effect due to the detector's recoil and the quantum smearing may have similar consequences for the Hawking effect.

Surprisingly, since the work of Unruh [2] two decades ago, this problem has attracted little attention. In his original paper [2], Unruh suggested a two-field model for a finite mass particle detector. Two scalar fields Ψ_M and $\chi_{M'}$, of masses M and $M' = M + \Omega$, respectively, were taken to represent two states of a detector. When the coupling to a scalar field ϕ is given by $\epsilon\phi\chi_{M'}\Psi_M$, "excitation" of this detector corresponds to a detection of a ϕ particle of energy Ω.

The two-field model was used recently by Parentani [4] to study the consequences of recoil and smearing on the Unruh effect. In this work two charged scalar fields where accelerated by means of a classical constant external electric field. The transition amplitude was obtained to first order in the coupling. Parentani showed that when the recoil and smearing are taken into account, up to Ω/M corrections, the transition amplitude is modified only by a phase. Therefore, to this order the thermal distribution and the Unruh temperature are unmodified. The new aspect that Parentani emphasized was the appearance of an additional phase in the transition amplitude which generates decoherence in successive emissions by the detector.

In this article we shall present an alternative first quantized treatment and rederive Parentani's result. We shall be studying the same problem of a finite mass particle detector in a constant electric field, but we use a different toy model. The accelerating detector is described as a first quantized, charged, point-like particle in a constant electric field with internal energy levels. A quantum version of (canonically conjugate) future and past Rindler horizon operators can be introduced [5], which facilitate the calculation of the transition amplitude and provides a simple intuitive physical picture of the recoil.

The model is then used to obtain the transition amplitude. In the zeroth order, neglecting Ω/M correction, we recover Parentani's result of an unmodified exact thermalization in the Unruh effect. The phase found in [4], that induces decoherence effects, is also obtained. In our approach, the phase is directly obtained by acting on the wave function with the horizon shift operator which induces the recoil.

The article is organized as follows. In Section 2. we introduce our model for an accelerated particle detector and construct the Rindler horizon operators. In Section 3, the transition amplitude is calculated and the result compared with that of ref. [4]. The recoil and quantum smearing are explicitly manifested in Section 4, and a qualitative simple picture of the recoil in terms of the shifting operators is demonstrated. In the following we adopt the units in which $\hbar = k_B = c = G = 1$.

II. ACCELERATED DETECTOR WITH FINITE MASS

In this section we present a model for a particle detector of finite mass which takes into account also the quantum nature of the detector's trajectory.

Consider a particle detector of rest mass M and charge q in a constant external electric field E_x in $1+1$ dimensions. Let us describe the internal structure by a harmonic oscillator with a coordinate η and frequency Ω. The internal oscillator is coupled to a free scalar field ϕ. The total effective action is

$$S = -M\int d\tau - qE_x\int X dt + \frac{1}{2}\int\Big(\Big(\frac{d\eta}{d\tau}\Big)^2 - \Omega^2\eta^2\Big)d\tau + \int g_0\eta\phi(X(t(\tau)), t(\tau))d\tau + S_F. \quad (1)$$

Here, τ is the proper time in the detector's rest frame, X is the position of the detector, g_0 is the coupling strength with a scalar field ϕ and S_F is the action of the field. Since we would like to describe the back reaction on the trajectory let us rewrite this action in terms of the inertial frame time t. The action of the accelerated detector is then given by

$$\int\Big[\Big(-M - g_0\eta\phi(X,t)\Big)\sqrt{1-\dot{X}^2} - qE_x X]dt + \frac{1}{2}\int\Big[\frac{1}{\sqrt{1-\dot{X}^2}}\Big(\frac{d\eta}{dt}\Big)^2 - \sqrt{1-\dot{X}^2}\Omega^2\eta^2\Big]dt. \quad (2)$$

This yields a simple expression for the Hamiltonian of the total system with respect to the inertial frame:

$$H = \sqrt{P^2 + M_{eff}^2} - qE_x X + H_F, \tag{3}$$

where the effective mass M_{eff} is given by

$$M_{eff} = M + \frac{1}{2}\Big(\pi_\eta^2 + \Omega\eta^2\Big) + g_0 \eta \phi(X), \tag{4}$$

and $\pi_\eta = \frac{\partial L}{\partial \dot{\eta}} = \dot{\eta}/\sqrt{1 - \dot{X}^2}$. The validity of our model rest upon a the assumption that the Schwinger pair creation effect can be neglected for our detector.

Since the Schwinger pair creation process is damped by the factor $\exp(-\pi M^2/qE_x)$ this implies the limitation $M^2 > qE_x$. Notice that since the acceleration is $a = qE_x/M$, this implies that $M > a = 2\pi T_U$. In the following we set $E_x = 1$ for convenience.

To obtain a quantum mechanical model we simply need to impose quantization conditions on the conjugate pairs X, P and η, π_{eta} and use the standard quantization procedure for the scalar field. It is convenient to introduce internal energy level raising and lowering operators $A^\dagger$ and A. The harmonic oscillator Hamiltonian can then be replaced by $\Omega A^\dagger A \equiv \Omega N$ and the internal coordinate by $\eta = i(A^\dagger - A)/\sqrt{2\Omega}$. This form can also be used in other, more general cases, however the simple commutation relation $[A, A^\dagger] = 1$ in the case of a harmonic oscillator, needs to be modified accordingly.

So far we have not imposed a limitation on the coupling strength g_0. In the case of small coupling $g_0(t) = \epsilon(t)$ the Hamiltonian can be written to first order in $\epsilon(t)$ as

$$H = H_D - qX + H_F + H_I. \tag{5}$$

Here

$$H_D = H_D(P, N) = \sqrt{P^2 + (M + \Omega A^\dagger A)^2} \tag{6}$$

is the free detector Hamiltonian, H_F is the free field Hamiltonian

$$H_F = \frac{1}{2}\int dx' [\Pi_\phi^2 + (\nabla\phi)^2 + m_f^2 \phi^2], \tag{7}$$

and

$$H_I = i\epsilon(t)\Big\{\frac{m_N}{H_D}, (A^\dagger - A)\phi(X,t)\Big\}, \tag{8}$$

where $m_N \equiv M + N\Omega = M + \Omega A^\dagger A$ and the anti-commutator, $\{A, B\} = \frac{1}{2}(AB + BA)$, maintains hermiticity. We have also absorbed a factor of $1/\sqrt{2\Omega}$ in the definition of $\epsilon(t)$. Comparing this interaction term with that used in the absence of a back-reaction we note that apart from the appearance of an anti-commutator there is also a new factor $\frac{m_N}{H_D}$. As we shall see, it corresponds to an operator boost factor from the inertial rest frame to the detector's rest frame.

In the Hiesenberg representation the eqs. of motion for the detector's coordinates X and P are given by:

$$\dot{X} = \frac{P}{H_D} - i\epsilon(t)\Big\{\frac{m_N P}{H_D^3}, (A^\dagger - A)\phi(X)\Big\}, \tag{9}$$

$$\dot{P} = q - i\epsilon(t)\Big\{\frac{m_N}{H_D}, (A^\dagger - A)\phi'(X,t)\Big\}, \tag{10}$$

where $\phi' = \frac{\partial\phi}{\partial x}$. We also have

$$\dot{A} = -i\big(H_{D,N+1} - H_{D,N}\big)A - i[A, H_I], \tag{11}$$

and

$$(\Box - m_f^2)\phi(x,t) = i\epsilon(t)\Big\{\frac{m_N}{H_D}, (A^\dagger - A)\delta(x - X)\Big\}. \tag{12}$$

In the zeroth order approximation ($\epsilon = 0$) the solution of eqs. (9-11) is

$$X^{(0)}(t) = X_0 + \frac{1}{q}\Big[H_D(t) - H_D(t_0)\Big], \qquad P^{(0)}(t) = P_0 + q(t - t_0), \tag{13}$$

$$H_D^{(0)}(t) = \sqrt{(P_0 + q(t - t_0))^2 + (M + \Omega A_0^\dagger A_0)^2}, \tag{14}$$

and

$$A^{(0)}(t) = \exp\Big[-i\int_{t_0}^{t}(H_{D,N_0+1} - H_{D,N_0})dt'\Big]A_0. \tag{15}$$

Here the subscript was used to denote the operator at time $t = t_0$ and the superscript to denote the zeroth order solution. To simplify notation we shall drop the superscript. Notice that $N_0 = A_0^\dagger A_0$ is a constant of motion in the zeroth order approximation.

It is now useful to introduce a proper time *operator* $\tau(t)$:

$$\tau(t) = \frac{M + \Omega A^\dagger A}{q} \sinh^{-1}\Big[\frac{q(t - t_0) + P_0}{M + \Omega A^\dagger A}\Big]. \tag{16}$$

The factor $(M + \Omega A^\dagger A)/H_D = m_N/H_D = \frac{d\tau}{dt}$ appearing in the coupling to the field (eq. 8) can therefore be interpreted as an operator boost factor $\frac{d\hat{\tau}(t)}{dt}$, from the inertial frame to the detector's rest frame. Notice that τ depends only on P_0 and N.

In terms of the proper time operator, the detector's trajectory can be simplified to:

$$t - t_0 - \tilde{T}_0 = \frac{1}{a} \sinh a\tau, \tag{17}$$

$$X - \tilde{X}_0 = \frac{1}{a} \cosh a\tau, \tag{18}$$

where

$$\tilde{T}_0 = -\frac{P_0}{q} \qquad \tilde{X}_0 = -\frac{H_D - qX}{q}, \tag{19}$$

and the acceleration a is given by the *operator*

$$a = a_N = \frac{q}{M + \Omega A^\dagger A} = \frac{q}{m_N}. \tag{20}$$

The operators $\tilde{T}_0$ and $\tilde{X}_0$ determine the location of the Rindler coordinate system of the detector with respect to the Minkowski coordinates (t, x). The space-time location of the intersection point of the future and past Rindler horizons is given by $(-t_0 - \tilde{T}_0, -\tilde{X}_0)$. Since

$$[\tilde{X}_0, \tilde{T}_0] = \frac{i}{q}, \tag{21}$$

the location of this space-time point becomes quantum mechanically smeared.

Another set of useful operators [5] we shall introduce is that of the location of the future and past Rindler horizons $\mathcal{H}_+$ and $\mathcal{H}_-$, respectively. They can be found from the relations

$$\mathcal{H}_+(t) = \lim_{t\to\infty} X(t), \qquad \mathcal{H}_-(t) = \lim_{t\to-\infty} X(t). \tag{22}$$

We find

$$\mathcal{H}_+(t) = -\tilde{T}_0 + \tilde{X}_0 + (t - t_0) = \frac{P(t)}{q} - \frac{H_D - qX}{q}, \tag{23}$$

and

$$\mathcal{H}_-(t) = \tilde{T}_0 + \tilde{X}_0 - (t - t_0) = -\frac{P(t)}{q} - \frac{H_D - qX}{q}, \tag{24}$$

Therefore we can express $X(t)$ as

$$X(t) = \mathcal{H}_+(t) + \frac{1}{a}e^{-a\tau} \quad \overset{t\to\infty}{\longrightarrow} \quad \mathcal{H}_+(t), \tag{25}$$

and

$$X(t) = \mathcal{H}_-(t) + \frac{1}{a}e^{a\tau} \quad \overset{t\to-\infty}{\longrightarrow} \quad \mathcal{H}_-(t). \tag{26}$$

In terms of $\mathcal{H}_\pm$, the Hamiltonian of the detector in an external electric field has the simple form:

$$H_D - qX = -\frac{q}{2}\Big(\mathcal{H}_+ + \mathcal{H}_-\Big). \tag{27}$$

Finally, $\mathcal{H}_\pm$ satisfy the commutation relation:

$$[\mathcal{H}_-, \mathcal{H}_+] = 2\frac{i}{q} \tag{28}$$

Examining eqs. (21) and (28), we notice that since $q = aM$, in the limit of constant acceleration but large mass, the commutators vanish as M^{-1} and the classical trajectory limit is restored.

III. THE TRANSITION AMPLITUDE

We shall now proceed to calculate the first order transition amplitude between the internal energy levels n and $n+1$ of the detector. To this end it will be most convenient to

use the interaction representation. The operators in this representation are the solutions of the free equations of motion given by (15,16,17,18), and the wave function satisfies the Schrödinger equation

$$i\partial_t|\Psi\rangle = H_I|\Psi\rangle. \tag{29}$$

Given at $t = t_0$ by the initial wave function $|\Psi_0\rangle$, to first order in ϵ the final state at time t is given by

$$|\Psi(t)\rangle = \Big[1 - i\int_{t_0}^{t} \epsilon(t')\Big\{\frac{m + \Omega A^\dagger A}{H_D}, i(A^\dagger - A)\phi(X, t')\Big\}dt'\Big]|\Psi(t_0)\rangle. \tag{30}$$

Let us set initial conditions for the internal oscillator to be in the n'th exited state $|n\rangle$, and for the scalar field to be in a Minkowski vacuum state $|0_M\rangle$. The initial state of the total system is therefore given by $|\Psi(t_0)\rangle = |0_M\rangle \otimes |n\rangle \otimes |\psi_D\rangle$, where $|\psi_D\rangle$ denotes the space component of the detector's wave function. Using the solution (15) for A and $A^\dagger$, the transition amplitude can be expressed as:

$$\begin{aligned}
|\Psi(t)\rangle = |\Psi(t_0)\rangle - \frac{\epsilon}{2}\int_{t_0}^{t} dt'\Big[\\
\sqrt{n+1}|n+1\rangle\Big(\frac{m_{n+1}}{H_{D,n+1}}e^{i\int_{t_0}^{t'}\Delta H_{n+1}dt''}\phi(X_n(t'), t') + e^{i\int_{t_0}^{t'}\Delta H_{n+1}dt''}\phi(X_n(t'), t')\frac{m_n}{H_{D,n}}\Big) \\
-\sqrt{n}|n-1\rangle\Big(\frac{m_{n-1}}{H_{D,n-1}}e^{-i\int_{t_0}^{t'}\Delta H_n dt''}\phi(X_n(t'), t') + e^{-i\int_{t_0}^{t'}\Delta H_n dt''}\phi(X_n(t'), t')\frac{m_n}{H_{D,n}}\Big) \\
\Big] \otimes |0_M\rangle \otimes |\psi_D\rangle.
\end{aligned} \tag{31}$$

Here we used the notation $\Delta H_n = H_{D,n} - H_{D,n-1}$. The subscript n (e.g. in $X_n(t')$), means that we need to substitute the free solutions with $N = n$. In two dimensions the solutions for a free massless scalar field can always be separated into right and left moving waves, i.e. $\phi = \phi_L(V) + \phi_R(U)$ where $U = t - x$, $V = t + X$. For simplicity we will limit the discussion to massless scalar fields and examine the solution only for right moving waves. Therefore, we substitute for ϕ:

$$\phi_R(U) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}\Big(e^{-i\omega U}a_\omega + e^{i\omega U}a_\omega^\dagger\Big). \tag{32}$$

Using eqs. (17,18,23) we find that on the trajectory of the detector the light cone coordinate U is given by

$$U|_D = t - X = -\mathcal{H}_{+0} - t_0 - \frac{1}{a} e^{-a\tau}. \tag{33}$$

Neglecting the constant phase factor $\exp(i\omega t_0)$, the final state can be written as:

$$|\Psi(t)\rangle = |\Psi(t_0)\rangle - \frac{\epsilon}{2}\int \frac{d\omega}{\sqrt{4\pi\omega}} \int_{t_0}^{t} dt' \Big[$$

$$\sqrt{n+1}|n+1\rangle \Big(\frac{m_{n+1}}{H_{D,n+1}} e^{i\int \Delta H_{n+1} dt''} e^{i\omega(-\mathcal{H}_{+0n} - \frac{1}{a_n} e^{-a_n\tau_n})} + e^{i\int \Delta H_{n+1} dt''} e^{i\omega(-\mathcal{H}_{+0n} - \frac{1}{a_n} e^{-a_n\tau_n})} \frac{m_n}{H_{D,n}} \Big)$$

$$-\sqrt{n}|n-1\rangle \Big(\frac{m_{n-1}}{H_{D,n-1}} e^{-i\int \Delta H_n dt''} e^{i\omega(-\mathcal{H}_{+0n} - \frac{1}{a_n} e^{-a_n\tau_n})} + e^{-i\int \Delta H_n dt''} e^{i\omega(-\mathcal{H}_{+0n} - \frac{1}{a_n} e^{-a_n\tau_n})} \frac{m_n}{H_{D,n}} \Big)$$

$$\Big] \otimes |1_{\omega M}\rangle \otimes |\psi_D\rangle. \tag{34}$$

This is an exact result in the first order approximation in ϵ. So far we have not introduced additional assumptions on M, Ω or $a_n = q/m_n$. We shall now apply a large mass limit. We shall assume that

$$M >> a_0 = \frac{q}{M}. \tag{35}$$

This restriction is indeed equivalent to a suppression of the Schwinger pair production process. Since for the Unruh radiation we need only detector energy gaps with $\Omega = O(a/2\pi)$, we also require

$$M >> \Omega, \tag{36}$$

Under these assumptions let us proceed to simplify expression (34).

First consider the term $\exp(\int \Delta H_{n+1} dt)$:

$$i\int_{t_0}^{t} \Delta H_{n+1} dt' = i\Omega \int_{t_0}^{t} \frac{m_n}{H_{D,n}} \Big[1 + \frac{1}{2}\frac{\Omega}{m_n}\frac{P^2}{H_{D,n}^2}\Big] + O(\Omega^3/M^3)$$

$$= i\Omega\tau_n + \frac{i}{2}\Omega^2 \Big[\frac{1}{m_n}\tau_n - \frac{1}{q}\tanh(a_n\tau_n)\Big] + c(P_0) + O(\Omega^3/M^3)$$

$$\simeq i\Omega\Big(\tau_n + \frac{1}{2}\frac{\Omega}{m_n}\tau_n - \frac{\Omega}{2q}(1 - 2\exp(-2a_n\tau_n))\Big) + c(P_0) + O(\Omega^3/M^3), \tag{37}$$

where $c(P_0)$ is a constant, and in the last line we have used the large τ approximation. This approximation is justified since the transition amplitude is dominated by contributions arising from integration over large τ. In the following we shall hence neglect the exponential correction and the constant $c(P_0)$ which gives rise only to an overall phase, and use the approximation:

$$i\int_{t_0}^{t}\Delta H_{n+1}dt' = i\Omega\tau_n\Big(1+\frac{1}{2}\frac{\Omega}{m_n}\Big). \tag{38}$$

Next consider the exponential terms in (34) which contain the horizon operator $\mathcal{H}_{+0}$. Only these terms maintain a dependence on the operator X as $\mathcal{H}_{+0} = X + G(P_0)$, where G is a function of P_0 only. Therefore, $[\mathcal{H}_+, P_0] = i$. Noting that the second term in the exponential depends only on P_0, and using the Campbell-Baker-Hansdorff identity yields:

$$\exp[-i\omega(\mathcal{H}_{+0}+\frac{1}{a_n}e^{-a_n\tau_n})] = \exp\Big[\frac{i}{2q}\omega^2 e^{-a_n\tau_n}\frac{m_n}{H_{D,n}} + O(M^{-2}a_0^{-2})\Big]\exp(-i\omega\frac{1}{a_n}e^{-a_n\tau_n})\exp(-i\omega\mathcal{H}_{+0}) \tag{39}$$

Notice that the unitary operator $e^{-i\omega\mathcal{H}_{+0}}$ generates the translation: $p_0 \to p_0+\omega$. This corresponds to a recoil of the detector and ensures total momentum conservation when the detector is exited and emits a Minkowski photon.

Finally, we consider the boost operator:

$$\frac{m_{n+1}}{H_{D,n+1}} = \frac{m_n}{H_{D,n}}\Big[1+\frac{\Omega}{m_n}\Big(1-\frac{m_n^2}{H_{D,n}^2}\Big)+O(\Omega^2/M^2)\Big]. \tag{40}$$

Since for large τ

$$\frac{m_n}{H_{D,n}} = \frac{1}{\cosh(a_n\tau_n)} = 2e^{-a_n\tau_n} - O(2e^{-3a_n\tau_n}), \tag{41}$$

we shall approximate this boost factor by

$$\frac{m_{n+1}}{H_{D,n+1}} = \frac{m_n}{H_{D,n}}\Big[1+\frac{\Omega}{m_n}\Big]. \tag{42}$$

We can now return to the transition amplitude (34) and for simplicity focus only on the amplitude $A(\omega, n+1, p) = \langle 1_\omega, n+1, p|\Psi(t)\rangle$. The indices $\omega, n+1, p$ correspond to

the outgoing states of the photon, internal detector levels and to the detector's momentum, respectively. Using eqs. (38,39,42) we find

$$A(\omega, n+1, p) = -\frac{i\epsilon}{2}\sqrt{\frac{n+1}{4\pi\omega}}\int_{t_0}^{t} dt'\Big[\Big(\frac{m_n}{H_{D,n}(p_0+\omega)} + \frac{m_n}{H_{D,n}(p_0)}\Big(1+\frac{\Omega}{m_n}\Big)\Big) \times \tag{43}$$
$$\exp\Big(i\Omega\Big(1+\frac{1}{2}\frac{\Omega}{m_n}\Big)\tau_n - i\omega\frac{1}{a_n}e^{-a_n\tau_n} + \frac{i}{2q}\omega^2 e^{-a\tau_n}\frac{m_n}{H_{D,n}}\Big)\Big]\phi_D(p+\omega).$$

Here, $\phi_D(p) = \langle p|\psi_D\rangle$. To obtain (43) we used a representation with $\mathcal{H}_{+0}$ and P_0 as conjugate operators, and used the unitary operator $\exp -i\omega\mathcal{H}_{+0}$ to generate translations in the momentum. At this point the transition amplitude is expressed as a c-number integral.

Let us proceed to investigate this integral. For large t the phase θ of the integrand can be approximated by

$$\theta = \Omega\Big(1+\frac{1}{2}\frac{\Omega}{m_n}\Big)\tau_n - \omega\frac{1}{a_n}e^{-a_n\tau_n} + \frac{1}{q}\omega^2 e^{-2a_n\tau_n}. \tag{44}$$

The stationary phase condition yield

$$\omega = -\Omega\Big(1 - \frac{\Omega}{m_n} + O(\Omega^2/M^2)\Big)\ e^{a_n\tau_n}. \tag{45}$$

This can be compared with the case of a classical trajectory obtained by sending $m \to \infty$. In the present case, the frequency at the stationary point is shifted by $\frac{\Omega^2}{m_n}e^{a_n\tau_n}$, which is in agreement with ref. [4] up to a numerical factor of 1/2. As long as $\frac{\Omega}{m_n} < 1$, the correction is small and the saddle point frequency remains exponentially high.

A second phase appears in the amplitude due to the shift in the momentum of the particle. Let the initial wave function of the detector be in an eigenstate of momentum, $|\psi_D\rangle = |k\rangle$. The horizon shift operator acting on the state yields

$$\begin{aligned} e^{-i\omega\mathcal{H}_{+0}}|k\rangle &= e^{-i\omega X_0}e^{i\omega T_0}e^{-i\omega^2/2q}|k\rangle \\ &= e^{-i(\omega k+\omega^2/2)/q}e^{-i\omega\tilde{X}_0}|k\rangle \\ &= e^{-i(\omega k+\omega^2/2)/q}|k-\omega\rangle \end{aligned} \tag{46}$$

where in the first line we have used the Campbell-Baker-Hausdorff identity. The phase, $(\omega k+\omega^2/2)/q$, is identical to that obtained by Parentani [4], (the factor $\omega^2/2$ arises here

from the non-commutativity of X_0 and T_0) . Here, the phase is directly obtained from the shift generated by the future horizon operator as a consequence of the recoil due to an emission of a scalar photon. This recoil is further discussed in Section IV.

Next consider the recoil affects on the boost factor $\frac{m_n}{H_{D,n}(p+\omega)}$ in eq. (43). The shift of $p \to p+\omega$ in this boost factor, is equivalent to a shift in time given by $t \to t' = t + \frac{\omega}{q}$. In terms of the proper time (which is now a c-number) this correspond to the transformation

$$\tau_n \quad \to \quad \tau_n' = \tau_n + \frac{\omega}{q} e^{-a_n \tau_n} \tag{47}$$

For transitions with $\tau_n(t) - \tau_n(t_0) >> 1/a_n$, this transformation does not modify the integral. Hence in terms of τ_n':

$$\frac{m_n}{H_{D,n}(p+\omega)} = \frac{d\tau_n'}{dt}. \tag{48}$$

The second, unshifted, boost factor can be expressed in terms of τ_n' as

$$\frac{m_n}{H_{D,n}}\Big(1 + \frac{\Omega}{m_n}\Big) = \frac{d\tau_n'}{dt}\Big(1 + \frac{1}{m_n}(\Omega + \omega e^{-a_n \tau_n'}) + O(\Omega^2/M^2)\Big). \tag{49}$$

Hence by expressing the integral (43) in terms of τ_n' we find that the two terms are equal up to order $O(\Omega^2/M^2)$ and an additional piece that (up to this order) vanishes at the stationary point (45).

Expressing the phase in terms of τ' we find

$$\theta = \Omega\Big(1 + \frac{1}{2}\frac{\Omega}{m_n}\Big)\tau_n' - \frac{\omega}{a_n}\Big(1 + \frac{\Omega}{m_n}\Big)e^{-a_n \tau_n'} + O(\Omega^2/M^2). \tag{50}$$

where the term involving $\frac{\omega^2}{q} e^{-2a_n \tau_n}$ in eq. (44) has dropped out and we are left only with the higher order corrections $O(\Omega^2/M^2)$, which will be neglected.

In terms of τ_n' the amplitude $A(\omega, n+1, p)$ can be written as:

$$-i\epsilon\sqrt{\frac{n+1}{4\pi\omega}}\phi_D(p+\omega)\Big[\int d\tau_n' \exp\Big(i\Omega\Big(1 + \frac{1}{2}\frac{\Omega}{m_n}\Big)\tau_n' - i\omega\frac{1}{a_n}\Big(1 + \frac{\Omega}{m_n}\Big)e^{-a_n \tau_n'}\Big) + \frac{\xi}{m_n}\Big] \tag{51}$$

where

$$\xi = \frac{1}{2}\int d\tau_n(\Omega + \omega e^{-a_n \tau_n}) \exp\Big(i\Omega\tau_n - i\frac{\omega}{a_n}(1 + \frac{\Omega}{m_n})e^{-a_n \tau_n}\Big) \tag{52}$$

For large τ_n, $\xi \sim O(\frac{\Omega}{M})$, and the term ξ/m_n can be neglected.

Finally we obtain:

$$A(\omega, n+1, p) = i\epsilon\sqrt{\frac{n+1}{4\pi\omega}}\phi_D(p+\omega)a_n^{-1}\Big(\frac{\omega}{a_n}(1+\frac{\Omega}{m_n})\Big)^{i\frac{\Omega'}{a_n}}\Gamma(-i\frac{\Omega'}{a_n})e^{-\pi\Omega'/2a_n} + O(\Omega^2/M^2), \tag{53}$$

where Γ is the Gamma function, and

$$\Omega' = \Omega\Big(1+\frac{1}{2}\frac{\Omega}{m_n}\Big) = \Omega\Big(1+\frac{1}{2}\frac{\Omega}{M}\Big) + O(\Omega^2/M^2). \tag{54}$$

This transition amplitude seems similar to the Unruh amplitude obtained in the absence of recoil and quantum smearing. Our amplitude agrees, (when Ω/M corrections are neglected), with the result obtained by Parentani [4]. As we have already seen, the momentum dependent phase relevant to the decoherence process [4], is also precisely obtained in our method (eq. 46).

IV. RECOIL AND QUANTUM SMEARING

The purpose of this section is to give a qualitative simple picture of the recoil. We shall show how this process can be simply expressed in terms of the effect of horizon shift operators on the detector's wave function. Since as we shall see, the recoil involves exponentially large shifts, in this section we can neglect the $1/M$ corrections.

Let us re-state the results of the last section in a more qualitative way. For the case of a classical trajectory, it was shown by Unruh and Wald [6] that if the detector is initially in the ground state then the final state can be written as

$$|\Psi(t)\rangle = |\Psi(0)\rangle - i|n=1\rangle \otimes a_{R\Omega}|0_M\rangle. \tag{55}$$

Here, $a_{R\Omega}$ is the annihilation operator of a quantum with frequency Ω with respect to the Rindler coordinate system that is defined by the detector's trajectory. Using the well known relation [2] of $a_{R\Omega}$ to Minkowski creation and annihilation operators a_M and $a_M^\dagger$, they get

$$|\Psi(t)\rangle = |\Psi(0)\rangle - iC(\Omega,a)|n=1\rangle \frac{e^{-\pi\Omega/2a}}{(e^{\pi\Omega/a}-e^{-\pi\Omega/a})^{1/2}} a_M^\dagger |0_M\rangle, \tag{56}$$

where C is a normalization factor. Note that $a_M^\dagger$ creates a positive frequency Minkowskian photon, which is not in a state of definite frequency ω. Qualitatively we can use the stationary phase approximation eq. (44) to relate the typical frequency of this photon to the time of emission τ.

We can now use the result obtained in the last section to replace eq. (56) with

$$\begin{aligned} |\Psi(t)\rangle &= |n=0,\psi_D,0_M\rangle \\ &- iC(\Omega,a_n)|n=1\rangle \frac{e^{-\pi\Omega/2a}}{(e^{\pi\Omega/a}-e^{-\pi\Omega/a})^{1/2}} \Big(e^{-iH_F\mathcal{H}_+} a_{MR}^\dagger + e^{+iH_F\mathcal{H}_-} a_{ML}^\dagger\Big)|0_M,\psi_D\rangle \end{aligned} \tag{57}$$

Here we have restored the full coupling with the left and right moving waves. The operators $a_{MR}^\dagger$ and $a_{ML}^\dagger$, correspond to creation operators of right and left moving waves respectively. This equation can be easily generalized to the case of transitions between any two levels n to $n+1$, as well as to the case of de-excitations. We have assumed that the scalar field is massless. However, for a massive field we simply need to replace $e^{-iH_f\mathcal{H}_{+0}}$ by $e^{iH_f\tilde{T}_0 - iP_f\tilde{X}_0}$ etc.

The new feature of eq. (57) is the insertion of the horizon shift operators $\exp(\pm iH_F\mathcal{H}_\pm)$ which act on the wave function of the detector and of the scalar field. These shift operators generate correlations between the "emitted" Minkowski scalar photon and the trajectory of the detector.

To illustrate these correlations, let us concentrate only on the left moving waves and express $a_{MR}^\dagger$ in terms of creation operators of definite Minkowski frequency:

$$a_{MR}^\dagger = \int f(\omega) a_\omega^\dagger d\omega \tag{58}$$

Eq. (57) can now be written as

$$|\delta\Psi\rangle = -iC'|n=1\rangle \int d\omega dh_+ e^{-i\omega h_+} f(\omega)\psi(h_+)|1_\omega\rangle \otimes |h_+\rangle. \tag{59}$$

Here we used a basis of $\mathcal{H}_{0+}$: $\mathcal{H}_+|h_+\rangle = h_+|h_+\rangle$. We see that the recoil interaction generates correlations between the shift h_+ in the u-time of the right moving "emitted" Minkowski

photons with the "horizons states" $|h_+\rangle$ of the detector. Therefore, the effect of "horizon smearing" yields, after emission, the final entangled state (59). In each component of this state, the Unruh effect is manifested, with the correction discussed in the previous section. Since the corrections do not depend on the uncertainty or the smearing Δh_+ of the future event horizon, the overall wave function still manifests the Unruh effect.

In order to examine the effect of the emission on the detector we can re-write eq. (59) by using as a basis the past horizon operator $\mathcal{H}_-$. We obtain:

$$|\delta\Psi\rangle = -iC'|n+1\rangle \int d\omega dh_- f(\omega)\psi(h_-)|1_\omega\rangle \otimes |h_- - \omega\rangle, \tag{60}$$

where $\psi(h_-) = \langle h_-|\Psi_D\rangle$. Since $\mathcal{H}_\pm$ are conjugate operators, the operator $\exp -i\omega\mathcal{H}_+$ has shifted the past horizon operator by ω. It is interesting to notice that the shift by ω of the past horizon can be exponentially large. In fact, from the stationary phase approximation we get that it is related to the time of emission τ as: $\Omega \simeq \Omega \exp(a\tau)$. Therefore a detection of a particle of energy Ω generates an exponential shift in the location of the past horizon of the detector:

$$\delta h_- = h_{-out} - h_{-in} \simeq \Omega \exp(a\tau) \tag{61}$$

The meaning of this shift is as follows. We can use the initial state ψ_{in} to define the location h_{-in} of the past horizon. We can also use the final state ψ_f of the detector and by propagating it to the past (with the free Hamiltonian) determine the location h_{-out}. These two locations differ by an exponential shift.

The propagation of a wave function to the past might seem strange. However the same phenomenon occurs if the detector is excited in the past at $\tau < 0$. In this case it emits a left moving Minkowski particle. We find that this induces an exponentially large shift in the location of the future event horizon operator $\mathcal{H}_+$:

$$\delta h_+ = h_{+out} - h_{-in} \simeq \Omega \exp(-a\tau) \tag{62}$$

The manifestation of the back reaction as an exponentially large shift is related to the method of 't Hooft [7] and of Schoutens, Verlinde and Verlinde [8]. In their case, infalling

matter into the black hole, induces an exponential shift of the time of emission of the Hawking photon in the future. The reason is that the Hawking photons stick so close to the horizon that even a small shift of the horizon still modifies the time of emission. In our case this exponential shift is related to the exponential energy of the emitted Minkowski photon. In both cases, the back reaction requires the existence of exponentially high frequencies in the vacuum. As in the case of Hawking radiation, a naive cutoff eliminates the thermal spectrum seen by the Unruh detector.

Acknowledgment

I wish to thank W. G. Unruh for discussions and very helpful comments. I have also benefited from discussions with S. Nussinov and J. Oppenheim.

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